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LONG-TERM MEMORY IN THE INDIAN FOREX MARKETS

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Abstract

This study explores long-term memory in Indian forex markets by analysing daily exchange rate returns for USD/INR, EUR/INR, GBP/INR and JPY/INR from 2000 to 2024. We evaluate the mean and volatility dynamics of these currency pairs using a variety of fractal models, including ARFIMA, FIGARCH, and APARCH-FIGARCH techniques. Stationarity tests suggest that all series are stationary, whereas Hurst exponent analysis shows persistent behaviour (H > 0.5) for USD/INR and JPY/INR, as well as near-random walk behaviour for EUR/INR and GBP/INR. The ARFIMA estimation for the mean gives no significant long-memory impact (d = 0) for certain pairs, while the FIGARCH and APARCH-FIGARCH models captured strong long-term persistence and asymmetric volatility. These findings imply that, while the efficient market theory holds that exchange rates follow a random walk, the presence of long memory, particularly in volatility, implying that the Fractal Market Hypothesis holds in the Indian Forex Markets. The findings reveal persistent and asymmetric volatility in INR currency pairs, aiding traders in refining hedging, VaR estimation, and option pricing models. For central banks, this suggests the need for timely and sustained interventions to ensure exchange rate stability.

Keywords: Indian Forex Markets, Fractional Integration, ARFIMA, ARMA, FIGARCH, FIAPARCH.

JEL Classification: C32, F31.

1. INTRODUCTION

According to the Bank for International Settlement (BIS) 2022 study, the foreign exchange (FX) market is the world's largest and most liquid financial market, with a daily trading volume of more than $7.5 trillion. The foreign exchange market is critical to international trade, investment, and monetary policy transmission. Macroeconomic causes, interest rate differentials, geopolitical developments, and speculative actions all contribute to its considerable volatility (Engle, 1982). Volatility in the forex market reflects the magnitude of exchange rate fluctuations over time, making it a key concern for traders, investors, and policymakers (Bollerslev, 1986).

High volatility can lead to sharp currency depreciation or appreciation, impacting import-export competitiveness, inflation, and economic stability. A critical characteristic of forex market volatility is its long-term memory, which refers to the persistence of price movements and dependence on past values (Hurst, 1957). Long memory in a time series indicates enduring temporal dependency in the data. The presence of long memory in a time series indicates that shock at one point does not fade fast. It continues in a deteriorating manner. This influences future outcomes (Kumar, 2014).

The Efficient Market Hypothesis (EMH) as proposed by the (Fama, 1970) suggests that asset prices fully reflect all available information, making it impossible to achieve consistent excess returns through technical or fundamental analysis. According to the EMH, FX prices should follow a random walk, which was first proposed by (Bachelier, 1990) with future price changes unaffected by previous values. However, empirical studies show that exchange rates frequently deviate from efficiency due to market frictions, behavioural biases, and structural breaks (Lo, 2004). This has led to the development of alternative theories such as the Fractal Market Hypothesis (FMH), which claims that financial markets operate over various time horizons and exhibit self-similar patterns (Edgar E. Peters, 1994). The prevalence of long-term memory in FX markets calls into question EMH assumptions, implying that price movements are impacted by underlying fractal structures and volatility clustering rather than being completely random.

Long memory components in financial markets cannot be fully explained by short memory systems. The short memory property refers to a series low-order correlation structure, where correlations between data at large lags are minimal (Kumar, 2014). There is a large body of literature on volatility modelling. Initially, continual volatility was assumed by the unconditional volatility model. Eventually, it was discovered that volatility changes throughout time, and shocks can last a long time.

Hence, traditional econometric models, such as the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models were developed by (Engle, 1982; Bollerslev, 1986), respectively. These models imply short-term dependence in volatility clustering. However, financial markets display long-range dependency and self-similarity, implying that previous volatility patterns influence future movements over long periods of time (B. Mandelbrot, 1963). (Granger & Joyeux, 1980) and (Hosking, 1981) developed the model with fractional difference in mean process known as the Autoregressive Fractionally Integrated Moving Average (ARFIMA). (Baillie et al., 1996) introduced the proposed a fractionally integrated Generalized Autoregressive Conditional Heteroskedasticity (or FIGARCH) model which introduces a fractional difference operator into the conditional variance equation, a further extension of which is called the Fractionally Integrated Asymmetric Power ARCH (or FIAPARCH) developed by (Tse, 1998) incorporates long memory, asymmetric effects, and power transformations of volatility in a time series data. (B. B. Mandelbrot, 1971) presents empirical evidence of long-term memory in asset values, showing that financial markets exhibit persistent dependencies and large, unpredictable fluctuations.

The study undermines the EMH by demonstrating that arbitrage does not always reduce inefficiencies, therefore supporting the FMH. These findings support fractal-based volatility models as a better alternative to classic GARCH models for capturing real-world market dynamics. The Indian foreign exchange market has evolved significantly since 1947. Initially, the rupee followed a par-value system (1947-1971) before shifting to a pegged exchange rate (1971-1975) and later to a basket of currencies (1975-1991). Post-1991 reforms introduced the Liberalized Exchange Rate Management System (LERMS), transitioning to a market-determined exchange rate in 1993. Since then, India has followed a managed float system, with the RBI intervening to stabilize volatility. Therefore, this paper aims to study the long-term memory effect of the following currency pairs USD/INR, EUR/INR, GBP/INR and JPY/INR by utilising the Fractal Models in capturing INR exchange rate effect.

Research Questions:

RQ1: Does the Indian Foreign Exchange market exhibits long-term memory in the mean process?

RQ2: Does the Indian Foreign Exchange market exhibits long-term memory in the variance process?

2. LITERATURE REVIEW

The concept of long-term memory in financial time series has been extensively studied, particularly in stock markets, commodities prices, and macroeconomic indicators. The presence of long memory means that past information continues to impact future price movements, calling into question the classic EMH (Fama, 1970). The FMH offers an alternative framework to EMH, suggesting that financial markets exhibit self-affine structures and long-range dependence(Blackledge & Lamphiere, 2022). While studies on stock markets have shown long-term memory in returns and volatility (Baillie et al., 1996), (Cont, 2001), less studies have focused on its prevalence in foreign exchange markets. Foreign currency markets are regarded as one of the most liquid and efficient financial markets due to their enormous trade volumes and decentralised trading processes. However, departures from the random walk theory have been discovered, indicating the possibility of long memory in exchange rate movements (Cheung, 1993). Long memory in forex markets affects market prediction, risk management, and trading methods. Given the scarcity of literature on long memory in forex markets, this review will draw ideas from studies on stock and commodity markets, as well as findings from broader financial time series research. The next sections will look at significant empirical research on long memory, the analytical approaches used to discover it, and the consequences for forex market efficiency.

The study of long memory in stock market volatility has received a lot of interest, especially with the use of fractionally integrated models, (Bollerslev & Mikkelsen, 1996) examined long-term dependencies in U.S. stock market volatility through fractionally integrated GARCH (FIGARCH) and exponentially weighted GARCH (EGARCH) models. Their study introduced a new class of flexible fractionally integrated EGARCH models, which effectively captured long-run volatility persistence in the Standard and Poor’s 500 composite index. Monte Carlo simulations demonstrated that these models provided a more accurate depiction of conditional variance, outperforming traditional ARCH models. The study also explored the implications of long memory in volatility forecasting and long-term option pricing, emphasizing the necessity of employing fractional integration in volatility modelling. Similarly, (Hiremath & Kumari, 2015) explored long memory in Indian stock market returns by analyzing daily values of 29 major indices, including sectoral indices from the National Stock Exchange and Bombay Stock Exchange, covering April 2003 to March 2012. Using multiple tests such as the Geweke and Porter-Hudak semiparametric test, Robinson’s Gaussian semiparametric test, and the Andrews and Guggenberger Bias Reduced test, the study found strong evidence of long memory in mid-cap, small-cap, and low-liquidity sectoral indices.

Conversely, large-cap indices exhibited mixed results. The findings suggested that market capitalization and liquidity influence long memory, aligning with prior research on emerging markets. Expanding on the market efficiency, (Arashi & Rounaghi, 2022) applied multifractal analysis to assess market efficiency and fractal characteristics of the NASDAQ stock exchange. Using ARMA-GARCH models to analyze daily returns from 2000 to 2016, the study determined that NASDAQ is an efficient and non-fractal market. Additionally, the ARMA-GARCH model demonstrated strong forecasting ability, with a 1% error margin in predicting daily returns for 2017. The findings reinforced the efficiency hypothesis for developed markets while contrasting with the persistent long memory observed in emerging markets. In addition to stock markets, the FMH has also been applied to cryptocurrencies by (Blackledge & Lamphiere, 2022) in risk estimation and market forecasting concluding that it is suitable for a long-term forecast, rather than being used for generating short-term predictions of actual price values.

The limited literature on long memory in forex markets though have provided valuable insights, (Kumar et al., 2017) tested the Fractal Market Hypothesis (FMH) across nine Asian Forex markets using a wavelet-based methodology. The study examined the impact of financial crises, particularly the 1997–1998 East Asian crisis and the 2008 global financial crisis, on Forex market behavior. The wavelet power spectra analysis revealed that both crises were driven by increased short-term trading activity, confirming that market fluctuations are influenced by trading horizons. The results supported the FMH, suggesting that financial instability arises when one time scale dominates market behavior. On similar lines, (Soofi et al., 2006) used the plug-in and Whittle methods, which rely on spectral regression analysis, to test for long memory in twelve Asian/dollar exchange rates. Their findings confirmed long memory in the Japanese Yen and Malaysian Ringgit, while other currencies exhibited short-memory processes.

The study highlighted the impact of structural breaks, particularly the 1997–1998 Asian financial crisis, on memory properties and suggested further research to account for these disruptions. (Diaz & Chen, 2017) used the ARFIMA-FIGARCH models to analyse nonlinearity in Forex returns in exchange-traded notes (ETNs). The findings support the presence of long-memory in currency volatility, contradicting the EMH's assumption of randomness. Furthermore, BDS and R/S analyses demonstrate deterministic chaos, which supports the notion that past price movements influence future volatility. (Mikhaylov, 2018) examines volatility spillover effects in emerging markets, including Forex. The findings show that long-memory features endure even in the face of external shocks like geopolitical crises and monetary policy changes. The use of FIGARCH models reveals that including structural breaks increases predicted accuracy.

Fractal study on Forex market anomalies (Kristjanpoller & Miranda Tabak, 2024) by reveals long-term dependencies. The study discovers that weekday effects influence persistence intensity, with various degrees of fractality across timeframes. The Hurst exponent research reveals that Forex markets have enduring trends, which allow traders to use momentum methods. (Kumar, 2014) conducted a study on the presence of long memory in the Indian Forex market using daily bilateral returns of the Indian Rupee against the US Dollar from 1994 to 2013. By employing ARFIMA-FIGARCH and ARFIMA-FIAPARCH models, the study confirmed long memory in both conditional mean and variance. The findings suggested that fractionally integrated models provide a superior fit compared to short-memory models, making them more effective for forecasting market volatility. The study also established that the Indian Forex market has a fractal structure, indicating persistence in volatility patterns. Aside from FX and equity markets, long memory has been seen in mutual funds and other financial time series, (Priyadarshini & Chandra Babu, 2012) analysed the persistence of trends and the presence of long-term memory in Indian financial time series using Rescaled Range (R/S) Analysis and Fractal Dimension Index (FDI). The data from the top 10 Indian mutual funds showed high persistence, meaning past trends strongly influence future movements. The results for Indian stock markets followed a biased random walk, meaning price movements are not entirely random but influenced by past trends.

While substantial study has been conducted on long memory in stock markets and volatility modelling, studies on FX markets are still very restricted. The extant literature shows persistent dependency in specific currency pairs, particularly in Asian and emerging economies, but thorough assessments are sparse. Given the growing importance of forex markets in global trade and investment, there is an obvious need for additional research into the persistence of exchange rate swings. This study seeks to fill that gap by examining long-term memory in the Indian forex market, revealing insights into its efficiency and volatility structure.

3. METHODOLOGY

To conduct the study, we will be utilising the ARFIMA – FIAPARCH Methodology. The data points considered for the study are daily data of the currency pairs, including USD/INR, EUR/INR, GBP/INR and JPY/INR, from 1/1/2000 to 31/12/2024.

3.1 Stationarity Test

The study to ensure the stationarity of the variables will conduct the Augmented Dickey Fuller (ADF) to check for the presence of a unit-root. Stationarity implies that the statistical properties of a time series, such as mean and variance, are constant over time. This stability is essential for making valid inferences about the data. For conducting the test for stationarity, we have used the natural log of the variables instead of absolute values.

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-1} + \epsilon_t \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

where:

- ϵ_t is the pure white noise error term
- ΔY_t is the first difference $(Y_{t-1} - Y_{t-2})$

$$H_0: \delta = 0$$

$$H_1: \delta < 0$$

In ADF Test we must prove that δ<0 i.e., reject H_0 indicating that the time-series is stationary.

3.2 Estimating the Hurst Exponent

The Hurst Exponent (H) as developed by (Hurst, 1957) is a statistical measure used to determine whether a time series has a tendency to persist (long memory). It is widely used in financial time series analysis to examine whether asset prices exhibit long-range dependence. We will be estimating the Hurst Exponent using the Rescaled Range (R/S) Analysis, The R/S statistic is based on the idea that the range of cumulative deviations from the mean, normalized by the standard deviation, follows a power-law relationship with time.

The R/S Statistics is given by:

$$E\frac{S(n)}{R(n)} = Cn^H \dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

where:

- $R(n)$ is the range of cumulative deviations from the mean over a window of size nnn:
- $S(n)$ is the standard deviation of the time series over the same window
- H Hurst Exponent, which determines the long-term memory of the series
- C is the constant

$$log\frac{R(n)}{S(n)} = \log C + H \log n \dots \dots \dots \dots \dots (3)$$

which suggests a linear relationship. The Hurst exponent H is estimated by performing a linear regression between $log\frac{R(n)}{S(n)} and \log n$

If $d > 0$, the series exhibits long-memory behaviour, and an ARFIMA model is appropriate, otherwise an ARMA model is appropriate.

3.3 Conditional Variance Test (ARCH LM Test)

The Conditional Variance Test is used to check for volatility clustering in a financial time series, meaning that large price movements tend to be followed by large movements and small ones by small movements.

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 \dots \dots \dots \dots \dots (4)$$

where:

- σ_t^2 : conditional variance at time t
- ω : constant term
- α : impact of past shocks (ARCH term)
- ϵ_{t-1}^2 : squared residual from the previous period

If we reject the Null Hypothesis i.e., No ARCH effects (variance is constant over time). We conclude that past squared residuals significantly influence current volatility and proceed with FIGARCH models.

3.4 Testing for Asymmetry in Volatility

The Sign Bias Test (Engle & NG, 1993) is used to detect asymmetric effects in volatility, meaning whether positive and negative shocks impact volatility differently.

$$\epsilon_t^2 = \alpha_0 + \alpha_1 S_t^- + \alpha_2 S_t^+ + \alpha_3 S_t^- \epsilon_{t-1} + v_t \dots \dots \dots \dots \dots .. (5)$$

where:

- ϵ_t^2 = squared residuals from a GARCH model
- S_t^- = 1 if $\epsilon_{t-1} < 0,$ negative shock
- S_t^+= 1 if $\epsilon_{t-1} > 0,$ positive shock
- $v_t =$ error term

$$\mathrm{H}_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$$

$$\mathrm{H}_1: \alpha_1 \neq \alpha_2 \neq \alpha_3 \neq 0$$

If the test rejects H_0, it suggests that negative shocks impact volatility differently than positive shocks, indicating the need for an asymmetric model like APARCH – FIGARCH

3.5 ARMA/ARFIMA – APRACH – FIGARCH Model

The Autoregressive Moving Average (ARMA) model is a fundamental time series model used to capture the dependencies in a stationary time series.

It combines two components:

- Autoregressive (AR) Component: Captures the relationship between a time series and its past values.
- Moving Average (MA) Component: Captures the relationship between a time series and past forecast errors (shocks).

The ARMA model is denoted as ARMA (p, q), where:

- p is the number of autoregressive lags.
- q is the number of moving average terms.

$$y_t = c + \sum_{i=1}^{p} \phi_i y_{t-1} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-1} + \varepsilon_t \dots\dots\dots\dots (6)$$

- y_t = observed value of the time series at time t
- c = constant term
- ϕ_i = autoregressive (AR) coefficients
- θ_j = moving average (MA) coefficients
- ε_t = white noise error term

The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model is used to analyse time series data that exhibit long memory or fractional integration.

Unlike traditional ARIMA models, which assume either stationarity or non-stationarity, ARFIMA allows for a fractional differencing parameter (d), capturing persistence in the data over long periods.

This model is particularly useful for financial time series where past values influence future values in a decaying but persistent manner.

$$\phi(B)(1-B)^d X_t = \theta(B)\epsilon_t \dots\dots\dots\dots (7)$$

where:

- X_t = time series at time t
- B = backward shift operator
- d = fractional differencing parameter
- $\phi(B)$ = autoregressive (AR) polynomial
- $\theta(B)$ = moving average (MA) polynomial
- ϵ_t = white noise error term

The APARCH model, introduced by (Ding et al., 1993), extends the standard GARCH model by, allowing for asymmetry in volatility (i.e., negative shocks may have a stronger impact than positive shocks) and introducing a power term to model different forms of heteroskedasticity.

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{p} \alpha_i(\mid \epsilon_{t-i} \mid -\gamma_i \epsilon_{t-1})^{\delta} + \sum_{j=1}^{q} \beta_j \sigma^{\delta} t - j \dots \dots \dots (8)$$

where:

- σ_t^{δ} = conditional volatility
- ω = constant term
- β = GARCH parameters capturing the persistence of past volatility.
- α = ARCH parameters capturing the impact of past squared shocks.
- γ = asymmetry parameter
- ϵ_t = shock term.
- δ = power transformation parameter, allowing for non-linear volatility responses.

The FIGARCH model, introduced by (Baillie et al., 1996) is an extension of the GARCH model that captures long-memory properties in volatility. Unlike GARCH, which assumes short memory, FIGARCH allows fractional integration, meaning past volatility shocks decay slowly over time.

$$\big(1-\beta(L)\big)(1-L)_t^d h = \omega + \alpha(\epsilon^2 t - 1) \dots \dots \dots (9)$$

- h_t = conditional volatility
- ω = constant term
- $\beta(L)$ = Lag polynomial of GARCH parameters
- α = ARCH parameters capturing the impact of past squared shocks.
- $(1-L)^d$ = Fractional differencing operator, capturing long-memory effects.

4. RESULTS AND INTERPRETATION

4.1 Stationarity Test (ADF Test)

Since all currency pairs have Tau-Statistics lower than Tau-Critical values and P-Values < 0.1, we reject the null hypothesis for all cases. This means that all four INR currency pairs (USD/INR, GBP/INR, EUR/INR, and JPY/INR) are stationary, meaning they do not exhibit unit root behaviour and do not require differencing to achieve stationarity.

Table 1: Augmented Dickey Fuller (ADF) Test for Stationarity

Currency Pair	Tau-Statistic	Tau-Critical	P-Value	Stationarity
USD/INR	-14.100532	-1.941041275	<0.01	Yes
GBP/INR	-83.447744	-1.941041073	<0.01	Yes
EUR/INR	-27.804266	-1.94103897	<0.01	Yes
JPY/INR	-15.695499	-1.941041282	<0.01	Yes

(Data Source: Investing.com, Author Generated in R Studio)

4.2 Hurst Exponent

GBP/INR shows mild mean-reverting tendencies, meaning price movements tend to revert to a long-term mean. EUR/INR behaves almost like a random walk, implying little predictability in movements. JPY/INR & USD/INR exhibit persistent trends, meaning past trends have some influence on future price movements. USD/INR is the most persistent, meaning it is likely to continue in its trend rather than reversing frequently. Therefore, for the currency pairs JPY/INR and USD/INR we can utilise the ARFIMA Model while for the GBP/INR and EUR/INR we will be utilise ARMA Model.

Table 2: Hurst Exponent

Currency_Pair	Hurst_Exponent
EUR.INR	0.503026777
GBP.INR	0.486024441
JPY.INR	0.530386236
USD.INR	0.536406674

(Data Source: Investing.com, Author Generated in R Studio)

4.3 Conditional Variance Test (ARCH LM Test)

The results from an ARCH test, which examines the presence of volatility clustering in financial time series. Significant ARCH effects present in JPY/INR which exhibits volatility clustering. Strong ARCH effects present, indicating substantial volatility clustering in the USD/INR exchange rate. For EUR/INR no ARCH effects detected implying volatility remains relatively stable over time.

No ARCH effects detected for GBP/INR it does not show signs of volatility clustering. Therefore, for the currency pairs JPY/INR and USD/INR we will utilise the FIGARCH model having inferred by the ARCH Model that there exists short-term memory. We will proceed with the FIGARCH Model for the remaining two currency pairs as well since the ARCH-LM test is designed to identify short-memory—i.e., whether recent shocks (over a few lags) significantly affect current volatility, however the FIGARCH captures long-run persistence in volatility, meaning past shocks decay very slowly over time, rather than disappearing quickly.

Table 3: Conditional Variance (ARCH LM Test)

Currency Pair	Chi-Squared	p-value	Decision
JPY/INR	630.16	<0.001	Reject H_0
USD/INR	977.15	<0.001	Reject H_0
EUR/INR	15.4648	0.116	Accept H_0
GBP/INR	8.9683	0.535	Accept H_0

(Data Source: Investing.com, Author Generated in R Studio)

4.4 Sign Bias Test (Asymmetry Volatility)

Across all currency pairs, negative shocks tend to increase volatility, while positive shocks decrease it. The effect is strongest for USD/INR, followed by JPY/INR, with GBP/INR and EUR/INR showing weaker but still significant asymmetry.

Persistence of past negative shocks is significant across all models, reinforcing the idea that past downturns contribute to stability. These results suggest asymmetry in volatility behaviour, supporting the presence of sign bias in currency movements, thus we can proceed with the APARCH-FIGARCH Model for all the currency pairs.

Table 4: Sign Bias Test for Asymmetric Volatility

USD/INR							JPY/INR					
Variable	**Estimate**	**Std. Error**	**t-value**	**p-value**	**Significance**		**Variable**	**Estimate**	**Std. Error**	**t-value**	**p-value**	**Significance**
Intercept	5.92E-14	0.1615	0	1	-		**Intercept**	0.74415	0.12143	6.128	9.40E-10	*** p < 0.001
S_neg	-1.201	0.1773	-6.773	1.37E-11	*** p < 0.001		**S_neg**	-0.38584	0.13326	-2.895	0.0038	** p < 0.01
S_pos	1.243	0.1688	7.363	2.02E-13	*** p < 0.001		**S_pos**	0.3465	0.1269	2.73	0.00634	** p < 0.01
S_neg_lag	-2.98	0.074	-40.261	< 2e-16	*** p < 0.001		**S_neg_lag**	-0.79103	0.05563	-14.219	< 2e-16	*** p < 0.001
GBP/INR							**EUR/INR**					
Variable	**Estimate**	**Std. Error**	**t-value**	**p-value**	**Significance**		**Variable**	**Estimate**	**Std. Error**	**t-value**	**p-value**	**Significance**
Intercept	0.81189	0.11797	6.882	6.45E-12	*** p < 0.001		**Intercept**	0.82351	0.09962	8.266	<2e-16	*** p < 0.001
S_neg	-0.25072	0.12947	-1.936	0.0529	. p < 0.1		**S_neg**	-0.23467	0.10933	-2.146	0.0319	* p < 0.05
S_pos	0.24752	0.12329	2.008	0.0447	* p < 0.05		**S_pos**	0.23278	0.10411	2.236	0.0254	* p < 0.05
S_neg_lag	-0.5444	0.05405	-10.072	< 2e-16	*** p < 0.001		S_**neg_lag**	-0.51521	0.04564	-11.288	<2e-16	*** p < 0.001

(Data Source: Investing.com, Author Generated in R Studio)

4.5 ARMA – APRACH – FIGARCH Model

We have used the ARMA process for all the currency pairs even though the Hurst Exponent for USD/INR and JPY/INR was greater than 0.5, indicating the utilisation of the ARFIMA Model. The Hurst exponent is a nonparametric measure that can detect long-range dependence, but it does not distinguish whether the long memory is in the mean or the volatility of a series. ARFIMA specifically models long memory in the mean process, via the fractional differencing parameter. If we fit an ARFIMA and find $d \approx 0$ (i.e., not statistically different from zero), it implies no long memory in the mean, even if $H > 0.5$, which is what we experienced while running the ARFIMA Model for the stated currency pairs. On the contrary, the FIGARCH Model show long-term memory in volatility, which could have inflated the value of the Hurst Exponent. The Hurst exponent is estimated via R/S analysis, which can be affected by structural breaks, volatility clustering, or trend components. When we run a parametric ARFIMA model, we specifically test whether the fractional differencing parameter is significantly greater than zero. If it isn't, we have no statistical evidence for long memory in the mean, which is what we experienced. Thus, we default to an ARMA specification, even if the nonparametric H was > 0.5.

Hence. for the EUR/INR currency pair, an ARMA (3,2) model was estimated with the following parameters: AR1 = -0.2677 (t = -1.97), AR2 = -0.7853 (t = -8.98), AR3 = -0.0696 (t = -4.93), MA1 = 0.2101 (t = 1.55), and MA2 = 0.7494 (t = 8.24). The strongly negative AR2 coefficient (–0.7853) suggests a notable corrective or oscillatory pattern in the returns, while the large positive MA2 coefficient (0.7494) indicates that shocks two periods ago have a significant impact on current values. AR1 and AR3 are also negative, although AR1 is only borderline significant. Overall, these results imply that EUR/INR exhibits short-memory dynamics driven by multiple lags, with both autoregressive and moving average components playing an important role.

For the GBP/INR currency pair, an ARMA (1,0) model (i.e., AR (1)) was fitted, yielding AR1 = -0.021 (t = -1.69). This coefficient is small in magnitude and only marginally significant, indicating that past values of GBP/INR have a minimal influence on current returns. In practical terms, the series behaves close to white noise in its mean, suggesting very limited predictability from past returns.

An ARMA (1,1) model was tested for JPY/INR, but both coefficients—AR1 = -0.0142 and MA1 = -0.0142—are extremely small (with relatively large standard errors). Such tiny estimates imply that past values and past shocks have negligible effects on the current returns. Consequently, JPY/INR appears nearly white noise in its mean equation, with no substantial short-memory patterns to exploit.

The USD/INR currency pair was modelled using an ARMA (5,1) specification, resulting in AR1 = -0.1747, AR2 = -0.0508, AR3 = -0.0229, AR4 = 0.046, AR5 = 0.0654, and MA1 = 0.1791. The moderately negative AR (1) term (–0.1747) indicates a slight corrective tendency from one period to the next, while the other AR terms are small but alternate in sign, reflecting a more complex short-memory structure compared to the other currency pairs. The MA (1) term (0.1791) is positive yet modest, suggesting that recent shocks do play a role, though not a dominant one. Overall, EUR/INR and USD/INR display more pronounced short-memory dynamics, whereas GBP/INR and JPY/INR behave closer to white noise in their mean equations, with minimal dependence on past shocks.

Table 5: Autoregressive (AR) Moving Average (MA) Model

EUR/INR				GBP/INR		
Parameter	**Estimate**	**Standard Error**		**Parameter**	**Estimate**	**Standard Error**
AR1	-0.2677	0.1356		AR1	-0.021	0.0124
AR2	-0.7853	0.0874				
AR3	-0.0696	0.0141		**USD/INR**		
MA1	0.2101	0.1355		**Parameter**	**Estimate**	**Std. Error**
MA2	0.7494	0.091		**ar1**	-0.1747	0.1684
				ar2	-0.0508	0.0126
JPY/INR				**ar3**	-0.0229	0.0154
Parameter	**Estimate**	**Std. Error**		**ar4**	0.046	0.0128
ar1	-0.0142	0.4529		**ar5**	0.0654	0.014
ma1	-0.0142	0.4528		**ma1**	0.1791	0.1686

(Data Source: Investing.com, Author Generated in R Studio)

The analysis of JPY/INR exchange rate volatility using the APARCH and FIGARCH models provides crucial insights into its persistence, asymmetry, and power transformation characteristics. The APARCH (1,1) model reveals that volatility exhibits significant persistence, as indicated by the high and significant β1 coefficient (0.9113), confirming that volatility shocks endure over time. Additionally, the negative and statistically significant asymmetry parameter (γ1 = -0.0869, p = 0.0147) suggests that negative shocks, such as JPY depreciation, lead to greater volatility than positive shocks of the same magnitude. The estimated power transformation parameter (δ = 2.722) highlights the flexibility in capturing volatility clustering, while the heavy-tailed conditional distribution underscores the presence of extreme fluctuations. In contrast, the FIGARCH (1,1) model emphasizes the long-memory property of volatility. The fractional integration parameter (δ = 0.999995) is highly significant, indicating that volatility follows a hyperbolic decay rather than the exponential decay seen in traditional GARCH models. This confirms that past volatility has a lasting influence on future volatility, a key characteristic of financial time series. While the short-term ARCH effect (α1 = 0.0412) is not statistically significant, the persistence parameter (β1 = 0.9468) remains highly significant, reinforcing the prolonged impact of past volatility.

Table 6: APARCH-FIGARCH JPY/INR

APARCH				
Parameter	**Estimate**	**Std. Error**	**t-value**	**p-value**
mu	-0.00014	0.00007	-2.06	0.0398
omega	0	0	0.0824	0.9343
alpha1	0.047655	0.014451	3.3	0.001
beta1	0.91132	0.030558	29.82	<0.0001
gamma1	-0.08689	0.035626	-2.44	0.0147
delta	2.722451	0.097946	27.8	<0.0001
shape	6.027367	0.303339	19.87	<0.0001

FIGARCH							
Parameter	**Estimate**	**Std. Error**	**t value**	**p-value**	**Robust Std. Error**	**Robust t value**	**Robust p-value**
mu	-0.00016	0.000071	-2.2919	0.021914	-0.00016	-2.3323	0.019686
omega	0	0	20.6057	0	0	3.5304	0.000415
alpha1	0.041191	0.030627	1.3449	0.178649	0.041191	0.8529	0.393727
beta1	0.946809	0.002121	446.3451	0	0.946809	82.5002	0
delta	0.999995	0.020503	48.7726	0	0.999995	19.6063	0
shape	5.563524	0.335443	16.5856	0	5.563524	15.9749	0

(Data Source: Investing.com, Author Generated in R Studio)

The APARCH (1,1) model results for the USD/INR exchange rate, the presence of a statistically significant negative asymmetry parameter ($\gamma 1 = -0.0635$, $p = 0.0414$) indicates that negative shocks have a stronger effect on volatility than positive ones of the same magnitude, suggesting a leverage effect. Additionally, the power transformation parameter ($\delta = 2.1468$) is highly significant, confirming that the conditional variance follows a nonlinear process. The high persistence of volatility is reflected in the estimated $\beta 1 = 0.9127$, suggesting that volatility clustering remains a key characteristic. The FIGARCH (1,1) model emphasizes long-term memory in volatility. The fractional differencing parameter ($\delta = 0.995$) is close to one, indicating that volatility shocks decay very slowly over time, supporting the presence of strong volatility persistence. This is further reinforced by the high autoregressive coefficient ($\beta 1 = 0.9238$), which suggests that past volatility significantly influences future volatility.

Table 7: APARCH-FIGARCH USD/INR

APARCH				
Parameter	**Estimate**	**Std. Error**	**t value**	**p-value**
mu	-1.9E-05	0.000018	-1.041	0.2979
omega	0	0	0.107	0.9148
alpha1	0.07835	0.006055	12.94	0
beta1	0.912664	0.007126	128.08	0
gamma1	-0.06353	0.031153	-2.039	0.0414
delta	2.146827	0.00751	285.873	0
shape	5.116212	0.264492	19.344	0
FIGARCH				
Parameter	**Estimate**	**Std. Error**	**t value**	**p-value**
mu	-1.5E-05	0.000018	-0.8225	0.4108
omega	0	0	0.32	0.749
alpha1	0.114626	0.02686	4.2676	0.00002
beta1	0.923806	0.003606	256.22	0
delta	0.995016	0.012727	78.182	0
shape	4.739062	0.197196	24.0322	0

(Data Source: Investing.com, Author Generated in R Studio)

The APARCH (1,1) model results for the GBP/INR exchange rate indicate limited evidence of asymmetric volatility effects. The gamma parameter, estimated at -0.031851, suggests a potential leverage effect, where negative shocks could have a different impact on volatility than positive ones. However, its statistical insignificance (p-value = 0.329269) indicates that this asymmetry is not strongly present in the exchange rate volatility.

The power transformation parameter, delta, (2.875062, p-value = 0.000), confirming that the conditional variance follows a heavy-tailed distribution, which is common in exchange rate movements. The persistence of volatility is high, with beta1 estimated at 0.886554, implying that volatility shocks have long-lasting effects but eventually decay. The FIGARCH (1,1) model results for GBP/INR provide strong evidence of long-term memory in volatility.

The fractional differencing parameter, beta1, is estimated at 0.921775, suggesting a slow mean-reverting process where volatility shocks persist over extended periods. The significance of the delta parameter (0.962577, p-value = 0.000) confirms a heavy-tailed distribution of conditional variance, further emphasizing the presence of long-term dependencies.

Table 8: APARCH-FIGARCH GBP/INR

APARCH				
Parameter	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
mu	0.000063	0.000064	0.986317	0.323978
ar1	-0.043913	0.012862	-3.41415	0.00064
omega	0	0	0.032943	0.97372
alpha1	0.061196	0.014691	4.16556	0.000031
beta1	0.886554	0.020286	43.70264	0
gamma1	-0.031851	0.032648	-0.97559	0.329269
delta	2.875062	0.08735	32.91414	0
shape	7.976295	0.793185	10.05603	0
FIGARCH				
Parameter	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
mu	0.000089	0.000063	1.3951	0.162985
ar1	-0.04377	0.013034	-3.3578	0.000786
omega	0	0	14.5147	0
alpha1	0.074175	0.032842	2.2586	0.02391
beta1	0.921775	0.003242	284.3174	0
delta	0.962577	0.021569	44.6288	0
shape	9.076623	0.968227	9.3745	0

(Data Source: Investing.com, Author Generated in R Studio)

The APARCH (1,1) infers negative and insignificant asymmetry parameter (γ_1 = -0.05114) suggests that negative shocks do not significantly impact volatility differently than positive shocks. This indicates that leverage effects are weak in the EUR/INR exchange rate. The power transformation parameter (δ = 2.7856) confirms that the conditional variance follows a non-linear structure, which allows for a flexible modelling of volatility dynamics. Additionally, the persistence in volatility is evident through the high and significant beta coefficient (β = 0.9062), indicating that past volatility has a lasting impact on future volatility.

Table 9: APARCH-FIGARCH EUR/INR

APARCH				
Parameter	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
mu	0.000036	0.000061	0.583575	0.559506
ar1	-0.07028	0.012626	-5.56663	0
omega	0	0	0.029832	0.976201
alpha1	0.051866	0.015518	3.342232	0.000831
beta1	0.906173	0.020574	44.04522	0
gamma1	-0.05114	0.033393	-1.53149	0.125649
delta	2.785597	0.066554	41.85488	0
shape	9.217121	1.107333	8.323716	0
FIGARCH				
Parameter	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
mu	0.000063	0.000062	1.0107	0.312167
ar1	-0.06048	0.012819	-4.7181	0.000002
omega	0	0	5.3796	0
alpha1	0.067209	0.027848	2.4134	0.015803
beta1	0.944924	0.002226	424.4293	0
delta	0.967976	0.018624	51.9749	0
shape	9.567816	1.0423	9.1795	0

(Data Source: Investing.com, Author Generated in R Studio)

In contrast, the FIGARCH (1,1) model provides strong evidence of long memory in volatility. The estimated beta coefficient (β = 0.9449) is significantly close to 1, implying that volatility shocks exhibit high persistence and decay slowly over time. The estimated fractional differencing parameter (δ = 0.9679) further supports the presence of long memory, highlighting that volatility is neither purely stationary nor unit-root nonstationary but rather follows a fractional integration process.

5. CONCLUSION

The empirical study of the Indian forex market shows that, despite the stationarity of exchange rate returns, volatility dynamics have strong long memory and asymmetry. The USD/INR and JPY/INR pairs have high Hurst exponents (0.5364 and 0.5304, respectively), and the APARCH-FIGARCH model outputs show persistent volatility and large asymmetric responses to negative shocks. In contrast, the EUR/INR and GBP/INR pairings, with Hurst exponents near 0.50 and 0.4860, demonstrate low long memory in the mean; yet, fractal properties in their volatility persists. These findings demonstrate that classic ARMA or even ARFIMA models may not adequately reflect the complex dynamics of the Indian forex markets, as long memory appears to be more apparent in volatility than in the mean process. As a result, including fractal-based volatility models provides a more complex explanation of market behaviour, contradicting the conventional assumptions of the Efficient Market Hypothesis. Future study should look at the effects of structural breaks and external shocks to help refine these models and improve forecasting accuracy in emerging market environments. The findings have significant implications, for traders the presence of long-term volatility, implies that volatility shocks decays slowly, which is crucial for hedging strategies, option pricing and stop-loss limits. The discovery of asymmetry (negative shock increase volatility more than positive ones) helps improve Value at Risk (VaR) estimates. For central banks, the long-term memory suggest that interventions may need to be prolonged or anticipatory, it also helps RBI in deciding when and how long to deploy tools like spot market intervention or forward guidance to manage currency stability.

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